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LRISK

It takes two: the break-even strike of a call option from joint physical and pricing density estimation

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Joint work with D. Madan, W. Schoutens & S. Hölst

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Outline

1. Concepts
2. Calculating Physical and Pricing densities
3. Data example

Introduction

What normal people see:

What Financial Engineers see:

$\mathcal{P}$-world

- Physical world
- Actual world in which payoffs are realized
- Physical density p estimates real probabilities

Expected Payoff (t=0)
$= \mathrm{e}^{-rT} \cdot \mathbb{E}_{\mathcal{P}}[\text{payoff}]$

$\mathcal{Q}$-world

- Pricing world
- Artificial setting under which one determines the price
- Pricing density q reflects price a representative agent is willing to pay

Price
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Under no-arbitrage condition

Introduction to $\mathcal{P}$ and $\mathcal{Q}$

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The European Call Option - Expected Payoff

- European call option on asset S with maturity T and strike K
- Payoff $= (S_T - K)^+$

Expected Payoff European Call (at time 0)

$$= e^{-rT} \cdot \mathbb{E}_{\mathcal{P}}[(S-K)^+]$$

$$= e^{-rT} \cdot \int_{-\infty}^{\infty} (x-K)^+ p(x) dx$$

risk-free rate r

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risk-free rate r

Do both worlds agree at some point?

European call option

For a fixed maturity T, determine the **break-even strike** K_T such that

$$e^{-rT} \cdot \mathbb{E}_{\mathcal{P}}[(S - K_T)^+] = e^{-rT} \cdot \mathbb{E}_{\mathcal{Q}}[(S - K_T)^+]$$

⟶ Expectations in both worlds are equal

⟶ Expected return of European call $= \dfrac{\text{Price} - \text{Expected Payoff}}{\text{Price}} := 0$

Objective(s)

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$\longrightarrow$ Efficient estimation of the physical density p and pricing density q is needed

Estimating Physical and Pricing densities

Traditional approach

Physical density

- Estimated based on historical data
  - backward looking
  - only one new observation each day

Pricing density

- Estimated based on option data
  - forward looking
  - a number of new observations each day
- Depending on an asset pricing model

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Physical density → room for improvement!

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Alternative Physical density estimation - Step 1

(Madan, Schoutens & Wang, 2020)

Step 1: Pricing density as U-shaped perturbation of physical density

Assumptions:

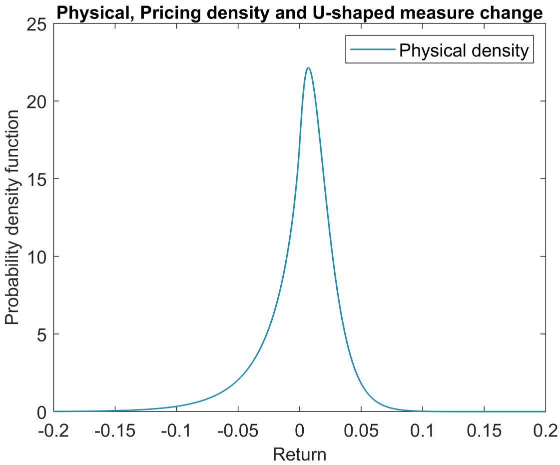
- Investors are risk-averse
- Investors have heterogeneous beliefs
  - long position is allowed
  - short position is allowed

Empirical evidence in e.g. (Bakshi et al., 2010)

Alternative Physical density estimation - Step 1

(Madan, Schoutens & Wang, 2020)

- **Long investor:** wealth loss in negative return state ⟶ loss protection leads to heavier left tail
- **Short investor:** wealth loss in positive return state ⟶ loss protection leads to heavier right tail



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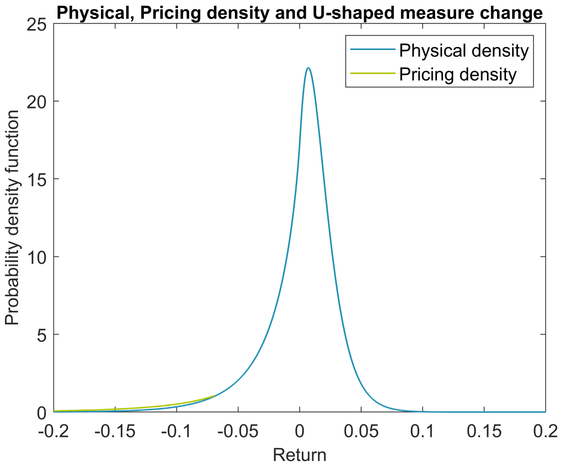

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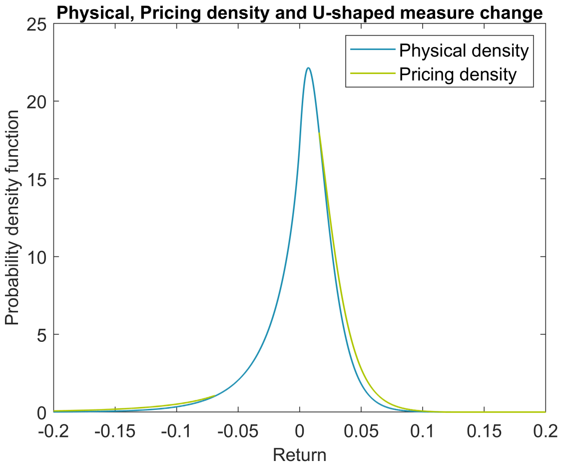

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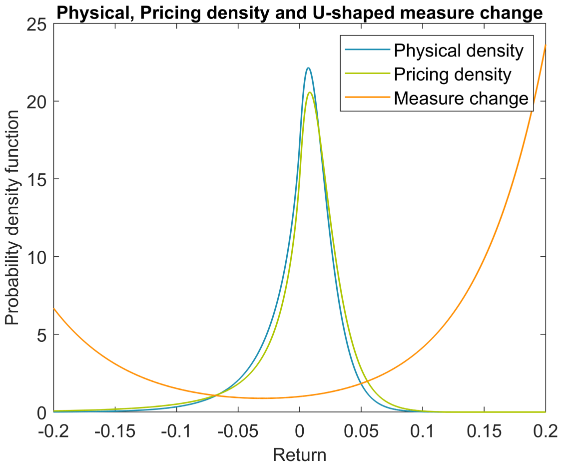

Alternative Physical density estimation - Step 1

(Madan, Schoutens & Wang, 2020)

Step 1: Pricing density as U-shaped perturbation of physical density

$$q(x) = C \cdot \left((1-\alpha) \cdot \mathrm{e}^{-\eta x} + \alpha \cdot \mathrm{e}^{\zeta x}\right) \cdot p(x)$$

- $C :=$ normalization constant
- $\eta :=$ risk-aversion coefficient for being in a long position
- $\zeta :=$ risk-aversion coefficient for being in a short position

Alternative Physical density estimation - Step 2

Step 2: Physical density follows a Bilateral Gamma model

- Bilateral Gamma (Küchler & Tappe, 2008) models the asset as

$$\log(S_t) = \log(S_0) + b_p \cdot \gamma_p(c_p t) - b_n \cdot \gamma_n(c_n t),$$

  where γ_p and γ_n are two independent standard Gamma processes

- Substantiated by different speed and scale for upward and downward movements of a stock (Madan & Wang, 2017)
  - Escalator up
  - Elevator down

Alternative Physical density estimation - Step 2

Step 2: Physical density follows a Bilateral Gamma model

$\cdot C \cdot \left((1-\alpha) \cdot e^{-\eta x} + \alpha \cdot e^{\zeta x}\right)$

Physical density p

- Bilateral Gamma
- Characterized by $[b_p, c_p, b_n, c_n]$

Pricing density q

- Tilted Bilateral Gamma
- Characterized by $[\eta, \zeta, \alpha, b_p, c_p, b_n, c_n]$

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from option data

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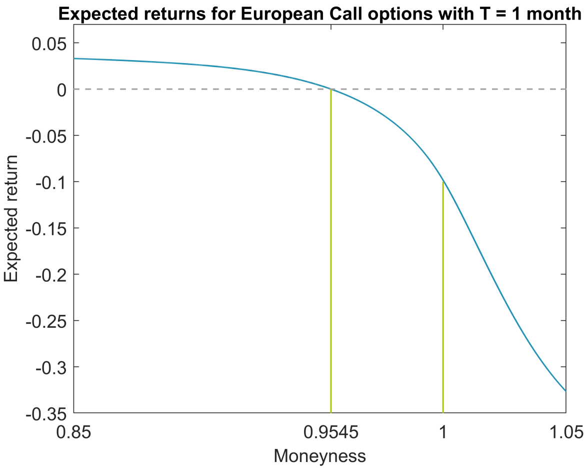
from option data

Expected return and break-even strike of a call option

illustrated based on S&P 500 index option data

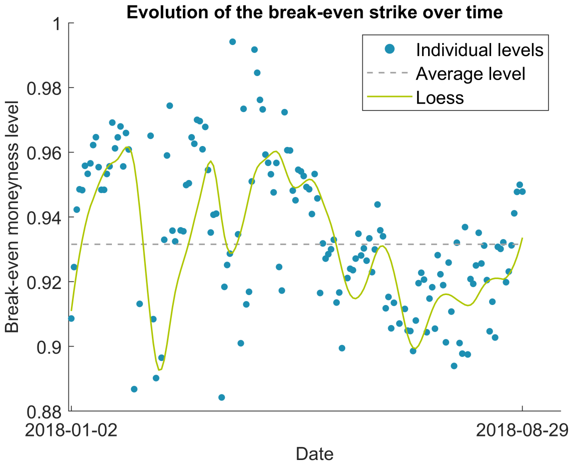
Expected Return European Call on S&P 500 index - March 15, 2018

- Fixed $T = 1$ month, $0.85 \leq \frac{K}{S_0} \leq 1.05$
- Expected return is decreasing with moneyness $\longrightarrow$ theoretical implication of U-shaped measure change (Bakshi et al., 2010)
- Break-even strike $K_T = 0.95 \cdot S_0$



Evolution of the Break-even Strike S&P 500 index

- Average break-even moneyness level of 93.15%
- Break-even strikes are located in-the-money
  - further away in-the-money call options are cheap
  - at-the-money and out-of-the-money call options are expensive
- Day-to-day fluctuations are small in absolute value



Findings and Conclusion

- The Tilted Bilateral Gamma model makes it possible to simultaneously estimate both physical and pricing density based on option data of the underlying asset
- This provides enough information to find the break-even strike of a call option

$\longrightarrow$ the data example shows a rather stable pattern over time

$\longrightarrow$ break-even strikes of S&P 500 index call options are in-the-money

Thank you!

References

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Additional info: Joint estimation of the densities

1. Find option data with prices of call and put options
2. Calibrate the Tilted Bilateral Gamma model parameters
   a. Calculate model prices of call options, $EC(K, T)$, with (Carr & Madan, 1998) formula

$$EC(K,T) = \frac{\exp(-\alpha \log(K))}{\pi} \int_0^\infty \exp(-\mathrm{i}\nu \log(K)) \varrho(\nu) d\nu,$$

   where

$$\varrho(\nu) = \frac{\exp(-rT)\mathbb{E}_{\mathcal{Q}}[\exp(\mathrm{i}(\nu - (\alpha+1)\mathrm{i}) \log(S_T))]}{\alpha^2 + \alpha - \nu^2 + \mathrm{i}(2\alpha+1)\nu}$$

   b. Minimize distance between model prices and market prices
3. Use an inverse Fourier transform to find the pricing density q and physical density p