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On the Pricing of Capped Volatility Swaps using Machine Learning Techniques

The Capped Volatility Swap

A capped volatility swap is a forward contract, with strike K, on an asset's annualized, realized volatility σ_R, over a fixed period of length T [2].

The **payoff** structure is given by

$$\text{Payoff} = \text{Notional} \times [\min(\text{Cap Level},\ \sigma_R) - K].$$

The Pricing Problem

At any time t, the **price** of a capped volatility swap is given by

$$\text{Price}_t = \text{DF}_t \times \mathbb{E}\,(\text{Payoff})\,,$$

with discount factor DF and expectations taken under a pricing measure.

Volatility swaps are **traded over-the-counter**, meaning that no price is readily available on exchange. The above equation is nonlinear, due to the cap level and the square root operator, which makes it a complex problem to solve.

An ML-based Solution

A model-free, data-driven approach to price capped volatility swaps, based on machine learning techniques, is explored.

Step 1 - Data

The data consists of time series of prices of multiple swap contracts on different underliers.

Response Variable - IVOL

$$\text{Price}_t = \text{DF}_t \times \left(\sqrt{\text{IVOL}_t^2 \times (1 - W_t) + \text{Accrued Vol}_t^2 \times W_t} - K \right)$$

Predictor Variables

	Model 1	Model 2
Accrued Volatility$_t$, Weight$_t$ (W_t), ITM*, K	✓	✓
Implied Volatility (IV)	✓	✓
30-day MA**(IV) - IV	✓	✓
Implied Skewness (IS)		✓
30-day MA(IS) - IS		✓

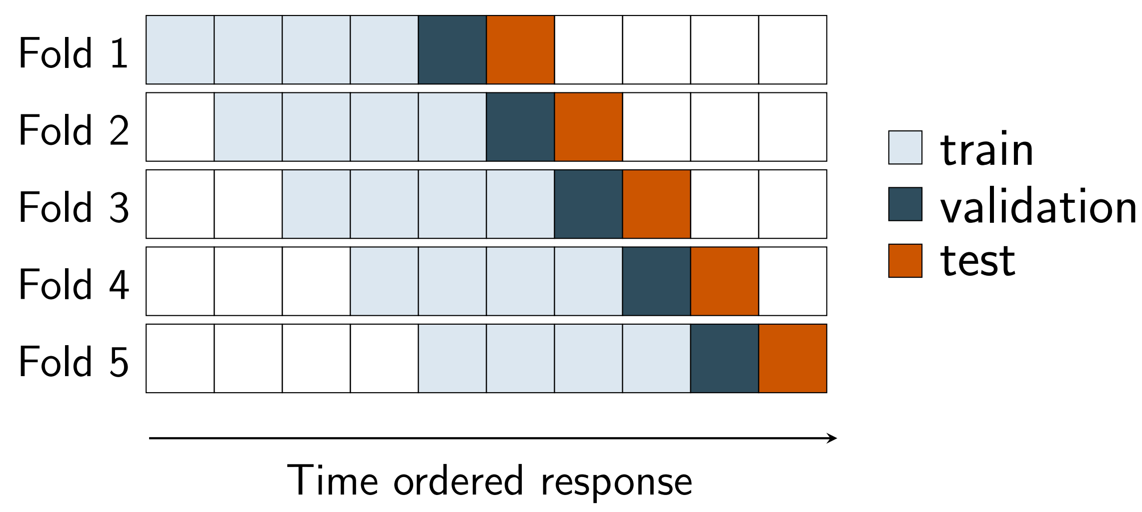
*ITM = Initial Time to Maturity

**MA = Moving Average

Market-implied volatility and skewness are estimated from quoted European vanilla option prices, using the model-independent method as explained in [4].

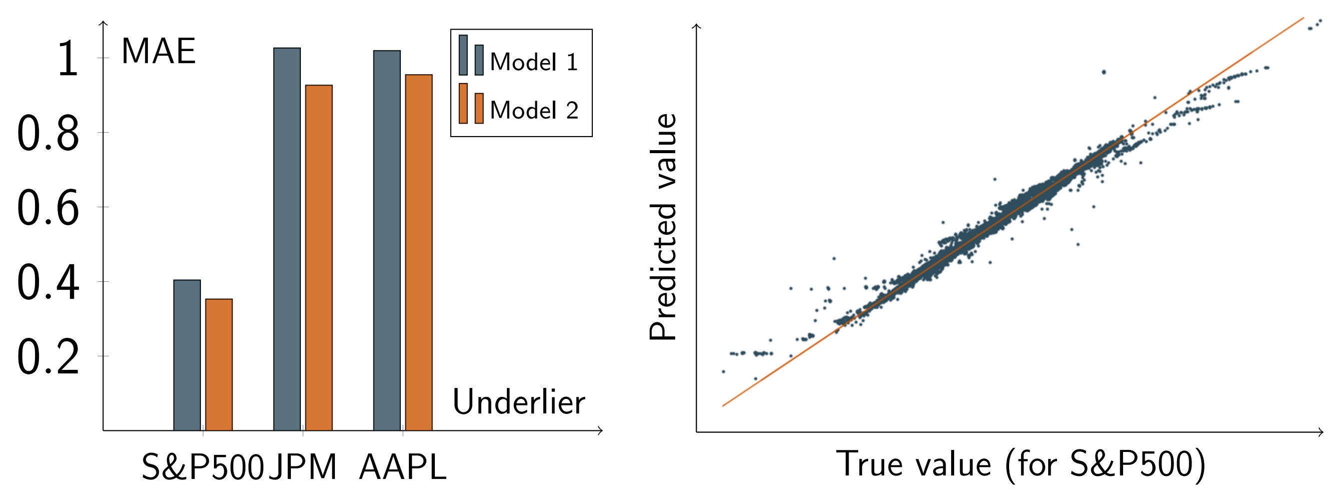
Step 2 - Modeling

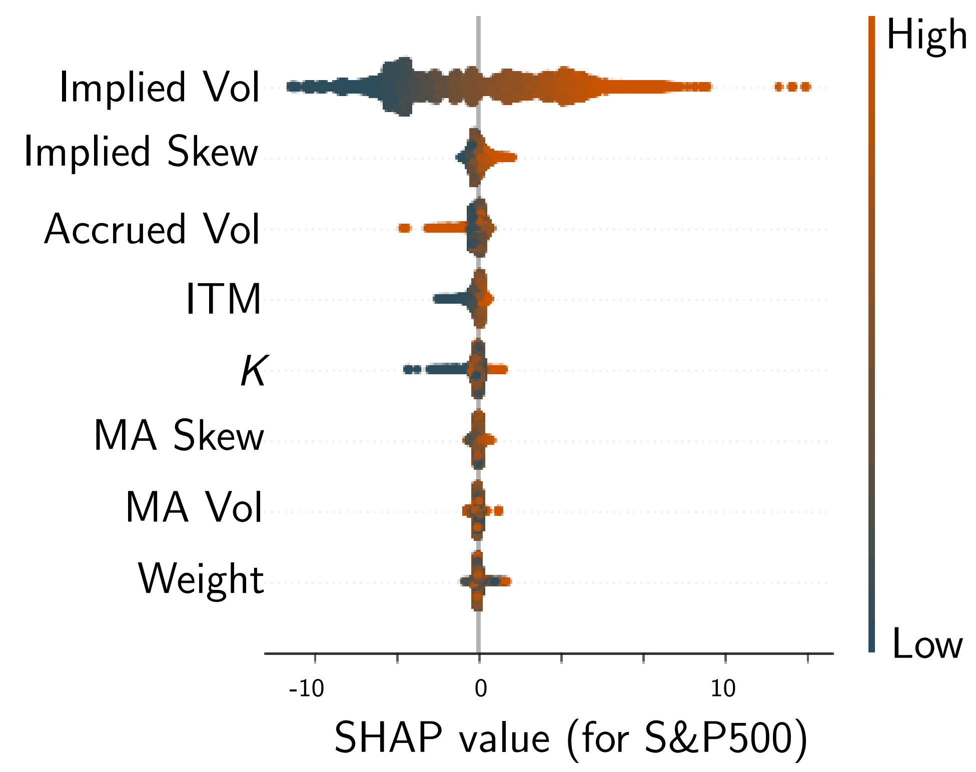
A Gradient Boosting Machine [3], using XGBoost, is trained to make predictions of IVOL. Hyperparameter tuning and model performance measurement are done using 5-fold, purged, walk-forward validation [1].



Step 3 - Results

The models are evaluated using the mean absolute error (MAE) of prediction. We show the average error over the 5 folds, for that part of the test set which has feature values within the training boundaries.





References

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